A.1 Construction of $Q^s(s'|\psi)$ and Proof of Lemma 1

Let $G \equiv \{0, 1/K, 2/K, \ldots \}$ be a uniform discrete approximation of $[0, 1]$. Let $\mathcal{S} = \{(s_1, s_2, \ldots s_B)|s_i \in G \text{ and } \sum_{i=1}^{B} s_i = 1\}$ be the associated probability simplex. Let $D = 1/K$ denote the distance between adjacent (grid) points of $G$.

**Lemma 4.** Let $s_i \in G$ for $i = 1, 2, \ldots B - 1$. If $\sum_{i=1}^{B-1} s_i < 1$, then $1 - \sum_{i=1}^{B-1} s_i \in G$.

**Proof.** $\sum_{i=1}^{B-1} s_i < 1 \Rightarrow \sum_{i=1}^{B-1} (\ell_i/K) < 1 \Rightarrow \sum_{i=1}^{B-1} \ell_i < K$ where the $\ell_i$'s are integers between 0 and $K$. Since a sum of integers is an integer and a difference of two integers is also an integer, $K - \sum_{i=1}^{B-1} \ell_i$ is a positive integer and it is less than $K$. Therefore, by the definition of $G$, $1 - \sum_{i=1}^{B-1} \ell_i/K \in G$. \hfill \Box

**Assumption 2.** All elements of the matrix $Q^\beta$ are strictly positive.

**Lemma 5.** Let $\psi = (\psi_1, \psi_2, \ldots, \psi_B)$ be any vector of type scores resulting from the Bayesian update. Then, $\psi_i \geq Q > 0$.

**Proof.** Let $Q$ be the smallest element of $Q^\beta$. By Assumption 1, $Q > 0$.

\[
\psi_i = \sum_j Q^\beta(i|j) \times \text{posterior probability of } j|\text{actions} \\
\geq \sum_j Q \times \text{posterior probability of } j|\text{actions} \\
= Q.
\]

The first equality follows from the definition of $\psi_i$, the inequality follows from Assumption 1 and the last line follows from the fact that the sum of posterior probabilities is 1. \hfill \Box

We now identify the elements of $\mathcal{S}$ that approximate any given type-score vector $\psi$ resulting from the Bayesian update. Let $s_{i,L} = \max_{s \in G} s \leq \psi_i$ and $s_{i,H} = s_{i,L} + D$. Consider the collection of $2^{B-1}$ vectors:

\[
S_\psi = \left\{(s_{1,l(1)}, s_{2,l(2)}, \ldots, 1 - \sum_{i=1}^{B-1} s_{i,l(i)}) \right\} \text{ where for each } i, l(i) \in \{L, H\}
\]

**Lemma 6.** If $D < Q/(B - 1)$ then $S_\psi \subset \mathcal{S}$.

**Proof.** By construction, $s_{i,L} \in G$. Next, observe that $s_{i,L}$ cannot be 1 since that would imply that $\psi_i = 1$ and, therefore, $\psi_{j \neq i} = 0$ in contradiction to Lemma 5. Therefore, $s_{i,H} = s_{i,L} + D$ must belong in $G$ for all $i$. To show that $(s_{1,l(1)}, s_{2,l(2)}, \ldots, 1 - \sum_{i=1}^{B-1} s_{i,l(i)})$ belongs in $\mathcal{S}$ it is sufficient
to show, by virtue of Lemma 4, that \( \sum_{i=1}^{B-1} s_{i,l(i)} < 1 \).

\[
\sum_{i=1}^{B-1} s_{i,l(i)} \leq \sum_{i=1}^{B-1} s_{i,H} \\
\leq \sum_{i=1}^{B-1} (\psi_i + D) \\
= (1 - \psi_B) + (B - 1)D \\
\leq 1 - Q + (B - 1)D \\
< 1
\]

The first inequality follows because \( s_{i,l(i)} \leq s_{i,H} \). The second inequality follows because \( s_{i,L} = s_{i,H} + D \) and \( \psi_i \geq s_{i,L} \). The third equality follows because \( \sum_{i=1}^{B} \psi_i = 1 \). The fourth inequality follows from Lemma 5 and the final inequality follows from the hypothesis of the Lemma.

By virtue of lemma 6 we can take \( S_\psi \) to be the collection of approximating vectors. Note that for each member of this set, the first \( B - 1 \) components are within \( \psi_i \pm D \) so, in this sense, the vectors are close to \( \psi \).

We now determine the probability assigned to each of these vectors. To this end, let

\[
p(s_{i,L}) = \frac{s_{i,H} - \psi_i}{D} \quad \text{and} \quad p(s_{i,H}) = \frac{\psi_i - s_{i,L}}{D} \quad \text{for} \quad i = 1, 2, 3, \ldots, B - 1
\]

Since \( s_{i,L} \leq \psi_i < s_{i,H} \) and \( s_{i,H} - s_{i,L} = D \) both \( p(s_{i,L}) \) and \( p(s_{i,H}) \) are nonnegative and their sum is 1. We set

\[
\Pr\left[\left( s_{1,l(1)}, s_{2,l(2)}, s_{3,l(3)}, \ldots, 1 - \sum_{i=1}^{B-1} s_{i,l(i)} \right)\right] = \prod_{i=1}^{B-1} p(s_{i,l(i)}), \quad l(i) \in \{L, H\}, \quad i = 1, 2, \ldots, B - 1.
\]

Then our assignment rule \( Q^*(s'|\psi) : S \to [0, 1] \) is given by:

\[
Q^*(s'|\psi) = \begin{cases} 
\prod_{i=1}^{B-1} p(s_{i,l(i)}) & \text{if} \ s' \in S_\psi \\
0 & \text{otherwise}
\end{cases}
\]

For this assignment rule, we can prove:

**Lemma 1.** (i) \( \sum_{s' \in S} s'_i Q^*(s'|\psi) = \psi_i, \forall i \), (ii) \( \sum_{s' \in S}(s'_i - \psi_i)^2 Q^*(s'|\psi) \leq 2(B - 1)D^2, \forall i \), and (iii) \( Q^*(s'|\psi) \) is continuous in \( \psi \).

**Proof.** (i) First, note that

\[
\sum_{s' \in S} s'_i Q^*(s'|\psi) = \sum_{s' \in S_\psi} s'_i Q^*(s'|\psi)
\]

since (26) assigns positive probability only to vectors that are in \( S_\psi \).

Let \( i \in \{1, 2, \ldots, B - 1\} \). Now, group the collection of vectors in \( S_\psi \) into two: In the first group are all vectors for which \( s'_i = s_{i,L} \) and in the second group are all vectors for which \( s'_i = s_{i,H} \). Denote
these groups as $S^L_\psi$ and $S^H_\psi$. Then,

$$\sum_{s'\in S^L_\psi} s'_i Q^s(s'|\psi) = \sum_{s'\in S^L_\psi} s'_i Q^s(s'|\psi) + \sum_{s'\in S^H_\psi} s'_i Q^s(s'|\psi)$$

$$= s_{i,L} \sum_{s'\in S^L_\psi} Q^s(s'|\psi) + s_{i,H} \sum_{s'\in S^H_\psi} Q^s(s'|\psi)$$

$$= s_{i,L} p(s_i,L) + s_{i,H} p(s_i,H)$$

$$= \psi_i$$

The third equality follows from the fact that the first and second summation terms in the second line are the probabilities of selecting a vector that belongs to group $L$ and group $H$, respectively. Since the assignment of $s_{i,L}$ or $s_{i,H}$ for $s'_i$ is done independently of the assignments to the other $B-2$ components, the probability of selecting a vector in group $L$ is $p(s_i,L)$ and in group $H$ is $p(s_i,H)$. The last equality follows from (25).

Next, let $i = B$. Then,

$$\sum_{s'\in S_\psi} s'_B Q^s(s'|\psi) = \sum_{s'\in S_\psi} [1 - s'_1 - s'_2 - \ldots - s'_{B-1}] Q^s(s'|\psi)$$

$$= \sum_{s'\in S_\psi} Q^s(s'|\psi) - \sum_{i=1}^{B-1} \sum_{s'\in S_\psi} s'_i Q^s(s'|\psi)$$

$$= 1 - \sum_{i=1}^{B-1} \psi_i$$

$$= \psi_B$$

(ii) Let $i \in \{1, 2, \ldots, B-1\}$.

$$\sum_{s'\in S_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) = \sum_{s'\in S^L_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) + \sum_{s'\in S^H_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi)$$

$$\leq D^2 \sum_{s'\in S^L_\psi} Q^s(s'|\psi) + D^2 \sum_{s'\in S^H_\psi} Q^s(s'|\psi)$$

$$= D^2 (p(s_{i,L}) + p(s_{i,H}))$$

$$= D^2.$$
Let \( i = B \). Then,

\[
\sum_{s' \in S_\psi} (s'_B - \psi_B)^2 Q^s(s'|\psi) = \sum_{s' \in S_\psi} \left( 1 - \sum_{i=1}^{B-1} s'_i - 1 + \sum_{i=1}^{B-1} \psi_i \right)^2 Q^s(s'|\psi)
\]

\[
= \sum_{s' \in S_\psi} \left( \sum_{i=1}^{B-1} (s'_i - \psi_i) \right)^2 Q^s(s'|\psi)
\]

\[
= \sum_{i=1}^{B-1} \sum_{s' \in S_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) + \text{expectations of cross product terms}
\]

\[
\leq (B - 1)D^2.
\]

The inequality in the final line follows from the bound on each of the variances and from the fact that the assignments of \( s'_i \) for \( i \in \{1, 2, \ldots, B-1\} \) are independent of each other so that the expectation of all the cross product terms are zero.

(iii) Let \( \psi^n \) be a sequence converging to \( \psi^* \). Consider first the case where \( \psi^n \notin G \). Then, for \( n > N \), \( N \) sufficiently large, \( \psi^n \in (s^*_i, L, s^*_i, H) \) and, so,

\[
p^n(s_{i,L}) = \frac{s_{i,H} - \psi^n_i}{D} \text{ and } p^n(s_{i,H}) = \frac{\psi^n_i - s_{i,L}}{D}.
\]

It follows that \( \lim_{n \to \infty} p^n(s_{i,L}) = p^*(s_{i,L}) \) and \( \lim_{n \to \infty} p^n(s_{i,H}) = p^*(s_{i,H}) \). Next consider the case where \( \psi^n \in G \). Then, by construction

\[
s_{i,L} = \psi^n, \quad s_{i,H} = s_{i,L} + D \text{ and } p^*(s_{i,L}) = 1.
\]

Then, for \( n > N \), \( N \) sufficiently large, either \( \psi^n \in (s_{i,L} - D, s_{i,L}) \) or \( \psi^n \in (s_{i,L}, s_{i,L} + D) \). Therefore, \( p^n(s_{i,L}) \) converges to 1 = \( p^*(s_{i,L}) \) as \( \psi^n \) converges to \( \psi^* \).

Note that by reducing the distance between adjacent points of \( G \), or, equivalently, increasing the number of (uniformly-placed) grid points approximating the unit interval, the dispersion of \( s' \) around \( \psi \) can be made arbitrarily small.

**A.2 Equivalence**

Let \( \Omega = \{ \mathcal{E} \times \mathcal{A} \times \mathcal{S} \} \) with typical element \( \omega = (e, a, s) \) and \( \hat{\Omega} = \{ \mathcal{E} \times \mathcal{A} \times \mathcal{P} \} \) with typical element \( \hat{\omega} = (e, a, m) \). Recall that \( m(\omega) \equiv p^{(0, \hat{d})}(\omega) = p^{(0, \hat{d})}(e, a, M(\hat{\omega})). \)

An individual who considers whether to default \( d \) or choose asset \( a' \) in state \( (e, \beta, z, \hat{\omega}) \) takes as given

- the price function \( q^{(0, a')}(\hat{\omega}) : \mathcal{A} \times \hat{\Omega} \rightarrow [0, 1] \),
- the credit-score transition function \( Q^m(m'| (d, a'), e') : \mathcal{P} \times \mathcal{A} \times \mathcal{E} \rightarrow [0, 1] \),

when optimizing. As in (2), this implies that an individual of type \( \beta \) in state \( (z, \hat{\omega}) \) chooses \( (d, a') \in \mathcal{F}(z, \hat{\omega}) \) inducing consumption \( c^{(d, a')}(z, \hat{\omega}) \) satisfying:

\[
c^{(d, a')}(z, \hat{\omega}) = \begin{cases} 
  e + z + a - q^{(0, a')}(\hat{\omega}) \cdot a' & \text{if } (d, a') = (0, a') \\
  e + z - \kappa & \text{if } a < 0 \text{ and } (d, a') = (1, 0)
\end{cases}
\]  

(27)
For all \((d,a') \in \mathcal{F}(z,\hat{\omega})\), the value functions given by equations (3), (5), (9), (11) and choice probabilities given by equations (6), (7), (8) associated with the individual’s problem are unchanged in form after substituting \(\hat{\omega}\) for \(\omega\) except for equation (4) now given by:

\[
v^{(d,a')}_{(\beta,z,\hat{\omega})} = (1 - \rho)u(c^{(d,a')}(z,\hat{\omega})) + \rho \cdot \sum_{(\beta',z',e',a',m')} Q^\beta(\beta'\mid \beta)Q^e(e'\mid e)H(z')Q^m(m'\mid (d,a'), e')W(\beta', z', (e', a', m'))
\]

(28)

Provided a positive measure set of credit contracts are issued, then prices in (13) are now given by :

\[
q^{(0,a')}_{(\omega)} = \begin{cases} \frac{p \cdot p^{(0,a')_{(e,a,M(e,a,m))}}}{1+r} & \text{if } a' < 0 \\ \frac{p}{1+r} & \text{if } a' \geq 0. \end{cases}
\]

(29)

To assess an individual’s probability \(p^{(0,a')}_{(\omega)}|f\) of repaying a debt next period given their current characteristics \(\omega\), the intermediary replaces the inference problem in (14) with

\[
\psi^{(d,a')}_{\beta'}(\omega) = \begin{cases} \sum_\beta Q^\beta(\beta'\mid \beta) \cdot \sum_{s'_{(\beta',z',e',a',m')}} \rho^s_{(\beta,\beta',z',e',a',m')} \cdot H(z')Q^e(e'\mid e) \cdot Q^s(s'|\psi^{(d,a')}_{\beta'}(\omega)) \cdot s'(\beta') \cdot \left(1 - \sigma^{(1,0)}(\beta', z', e', a', m(e', a', s'(\beta')))\right) & \text{for } (d,a') \in \mathcal{F}(z, e, a, m(\omega)) \\ \sum_\beta Q^\beta(\beta'\mid \beta) \cdot s'(\beta) & \text{for } (d,a') \notin \mathcal{F}(z, e, a, m(\omega)). \end{cases}
\]

(30)

Then, the probability of repayment in (15) that the intermediary uses to price debt and construct credit scores is now given by:

\[
p^{(0,a')}_{(\omega)} = \sum_{\beta',z',e',a',m'} H(z') \cdot Q^e(e'\mid e) \cdot Q^s(s'|\psi^{(d,a')}_{\beta'}(\omega)) \cdot s'(\beta') \cdot \left(1 - \sigma^{(1,0)}(\beta', z', e', a', m(e', a', s'(\beta')))\right).
\]

(31)

The transition function in equation (16) which tracks the probability that an individual in state \((\beta, z, e, a, m(\omega))\) transits to state \((\beta', z', e', a', m(\omega'))\) is now given by:

\[
T(\beta', z', \omega'; \beta, z, \omega) = \rho \cdot Q^\beta(\beta'\mid \beta) \cdot H(z') \cdot Q^e(e'\mid e) \cdot \sigma^{(d,a')}_{(\beta, z, m(\omega))} \cdot Q^s(s'|\psi^{(d,a')}_{\beta'}(\omega)) + (1 - \rho) \cdot G_{\beta}(\beta') \cdot H(z') \cdot G_{e}(e') \cdot 1_{\{a'=0\}} \cdot 1_{\{s'=G_{\beta}\}}.
\]

(32)

We can now give the definition of a stationary recursive credit-scoring competitive equilibrium.

**Definition 6. Stationary recursive credit-scoring competitive equilibrium:** A stationary recursive credit-scoring competitive equilibrium is a pricing function \(q^{(0,a')}_{*(\omega)}\), a type scoring function \(\psi^{(d,a')}_{*(\omega)}\), a choice probability function \(\sigma^{(d,a')}_{*(\omega)}\), a steady state distribution \(\mathcal{M}^*(\omega)\), and a credit score to type score mapping \(M^*(\omega)\) such that:

(i). \(\sigma^{(d,a')}_{*(\omega)}\) satisfies (6) and (7) for all \((\beta, z, \hat{\omega}) \in \mathcal{B} \times \mathcal{Z} \times \hat{\Omega}\) and \((d,a') \in \mathcal{F}(z, \hat{\omega})\),

(ii). \(q^{(0,a')}_{*(\omega)}\) satisfies (29) with equality for all \(\hat{\omega} \in \hat{\Omega}\) and \((d,a') \in \mathcal{F}(z, \hat{\omega})\) with \(p^{(0,a')}_{*(\omega)}\) satisfying (31) for all \(\omega \in \Omega\) and \((d,a' < 0) \in \mathcal{F}(z, (e, a, m(\omega)))\),

(iii). \(\psi^{(d,a')}_{*(\omega)}\) satisfies (30) for all \((\beta', \omega) \in \mathcal{B} \times \Omega\), and

(iv). \(\mathcal{M}^*(\beta, z, \omega)\) solves (17) for \(T(\beta', z', \omega'; \beta, z, \omega)\) in (32).
(v). $Q^m$ in (28) satisfies $Q^m(m' = \hat{m}((d,a'), e')) = Q^*(s' = M^*(e', a', \hat{m})|\psi^{(d,a')}(e, a, s))$ where $\hat{m} = p^{(0,a^*)}(e, a, M^*(e, a, \hat{m}))$ for any $\hat{m} \in \mathcal{P}$.

**Theorem 3.** Equivalence: For an individual in state $(\beta, z, e, a, s)$ in a RCE, let $(\beta, z, e, a, \hat{m})$ be the corresponding state in RCECS, where $\hat{m} = p^{(0,a^*)}(e, a, s)$. If the inverse function $M$ exists, an equivalent RCECS also exists in which the choice probabilities $\sigma^{(d,a')}(\beta, z, e, a, \hat{m}) = \sigma^{(d,a')}(\beta, z, e, a, s)$.

**Proof.** Assume (v) holds. Then provided $q^{(0,a^*)}(\omega) = q^{(0,a^*)}(\hat{\omega})$, $\mathcal{F}(z, \omega) = \mathcal{F}(z, \hat{\omega})$ in (2) and (27). By (v) (i.e. $Q^m(m' = \hat{m}((d,a'), e')) = Q^*(s' = M^*(e', a', \hat{m})|\psi^{(d,a')}(e, a, s))$, then $v^{(d,a')}(\beta, z, \omega)$ in (4) is identical to $v^{(d,a')}(\beta, z, \hat{\omega})$ in (28) as well as all other value functions. Then $\sigma^{(d,a')}(\beta, z, \omega) = \sigma^{(d,a')}(\beta, z, \hat{\omega})$ satisfying (i). If the choice probabilities are the same, then repayment probabilities in (15) and (31) are the same and prices in (13) and (29) are the same given the existence of the inverse function $M$, thus satisfying (ii). If prices are the same, then the conjecture that $\mathcal{F}(z, \omega) = \mathcal{F}(z, \hat{\omega})$ is verified. (iii) and (iv) hold since they are defined on $\omega$ (i.e. “in-house”).

**B Quantitative Appendix**

**B.1 Computational algorithm**

In this section, we describe the algorithm used to compute the baseline model presented in this paper. Note that the model is calibrated by using the procedure below to solve the model for a given set of parameters, and then updating these parameters to minimize the distance between the model moments and the data moments.

1. Set parameters and tolerances for convergence and create grids for $(\beta, e, z, a, s)$. Denote the length of these grids by $n_\beta, n_e, n_z, n_a$, and $n_s$ respectively.

2. Initialize the following equilibrium objects with sensible initial conditions:

   (a) value function, $W(\beta, e, z, a, s)$ from (11)
   
   (b) type scoring function, $\psi^{(d,a')}(e, a, s)$ from (14)
   
   (c) repayment probability, $p^{(0,a^*)}(e, a, s)$ from (15). Note that this implies a loan price schedule given condition (13).
   
   (d) stationary distribution, $\mu(\beta, e, z, a, s)$ from (17)

3. Taking the current guess of the equilibrium functions $f_0 = \{q_0, \psi_0\}$ as given, enter the equilibrium computation loop:

   (a) Solve for the expected value function $W_1(\cdot|f_0)$ taking as given $W_0(\cdot|f_0)$:
   
   i. Assess budget feasibility, finding the set of feasible actions $\mathcal{F}(e, z, a, s|f_0)$.
   
   ii. Compute the conditional value associated with each action $(d,a') \in \mathcal{F}(e, z, a, s|f_0)$, $v^{(d,a')}_1(\beta, e, z, a, s|f_0)$, according to (4), with $W(\cdot) = W_0(\cdot)$
   
   iii. Having looped over all feasible actions, aggregate the conditional values $v^{(d,a')}_1(\cdot|f_0)$ into the new expected value function $W_1(\cdot|f_0)$ according to (3).
   
   iv. Assess value function convergence in terms of the sup norm metric, $\sup |W_1(\cdot|f_0) - W_0(\cdot|f_0)|$. If less than tolerance, go to 3.b; otherwise, set $W_0(\cdot|f_0) = W_1(\cdot|f_0)$ and go back to 3.a.ii.

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33Since the full information version of the model solves very quickly, the value functions, loan price schedules, and stationary distributions from this case are good initial conditions.
(b) Compute the decision probabilities \( \sigma^{d,a'}(\beta, e, z, a, s|f_0) \) implied by \( W_1(\cdot|f_0) \) according to (6), (7), and (9).

(c) Given the decision probabilities \( \sigma(\cdot|f_0) \), compute the new set of equilibrium functions, \( f_1 = \{ q_1, \psi_1 \} \):
   i. Compute \( \psi_1^{d,a'}(e, a, s) \) according to (14).
   ii. Compute \( q_1^{0,a'}(e, a, s) \) according to (13).

(d) Assess equilibrium function convergence in terms of the sup norm metric
   \[
   \max \{ \sup |\psi_1 - \psi_0|, \sup |q_1 - q_0| \}
   \]
   If less than tolerance, proceed to step 4; otherwise, set \( f_0 = f_1 \) and go back to the beginning of step 3.

4. Compute the stationary distribution associated with the equilibrium behavior and the equilibrium functions computed in step 3.
   (a) For each state, compute \( \mu_1(\beta, e, z, a, s) \) according to (17), with \( \mu(\cdot) = \mu_0(\cdot) \) and given the set of equilibrium functions solved for above.
   (b) Assess convergence based on the sup norm metric \( \sup |\mu_1(\cdot) - \mu_0(\cdot)| \). If less than tolerance, go to step 5; otherwise, set \( \mu_0 = \mu_1 \) and go back to 4.a.

5. Compute moments:
   (a) aggregate credit market statistics: compute directly from stationary distribution
   (b) lifecycle statistics: simulate a cohort of individuals for a large number of periods, tracking the distribution over individual states at each age. Note that the initial distribution must be consistent with the assumptions on newborns from the main text.

B.2 The Role of Extreme Value Preference Shocks

One of the key modifications in our model relative to standard consumer bankruptcy models in macroeconomics is the inclusion of the additive, action-specific preference shocks. The mean of these extreme value shocks is irrelevant, but we calibrate the scale parameter \( \alpha \), which governs the overall variance of the shocks, as well as the correlation parameter \( \lambda \), which governs the correlation between the shocks associated with non-default actions. How does behavior in the model change with respect to these parameters? In this section we address this question in two ways. First, we use an analytical approach to show how these parameters directly affect choice probabilities, taking the value of specific actions (i.e. all equilibrium functions) and earnings processes as given. Then, in the context of the full information version of the model, we compute actual decision rules under different parameter combinations and describe the differences graphically.\(^{34}\)

B.2.1 Deriving the impact of EV parameters

To ease notation in this section, let an agent’s entire state be denoted by \( x = (\beta, e, z, a, s) \), and the set of feasible actions for that agent be denoted by \( F(x) \). The goal of this section is to show how the choice probability function \( \sigma \) varies with the extreme value scale and correlation parameters, \( \alpha \) and \( \lambda \). We first cover the non-default actions, and then default.

\(^{34}\)We choose the full information model purely for convenience, since the reduction in the state space reduces computation time and the number of states we need to account for in showing results. The same insights hold for other model variants.
Non-default actions  Equation (7) describes the probability of choosing a feasible action \((0, a')\) conditional on not defaulting. To ease computations in this section rather than compute derivatives with respect to \(\alpha\) or \(\lambda\) directly, we will compute them with respect to \(1/\alpha\) and \(1/\lambda\).\(^{35}\) Considering first \(\alpha\),

\[
\frac{\partial \hat{\sigma}^{(0,a')} (x)}{\partial (1/\alpha)} = \left\{ \exp \left\{ v^{(0,a')} (x)/\lambda \alpha \right\} \frac{v^{(0,a')} (x)}{\lambda} \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \exp \left\{ v^{(0,\tilde{a})} (x)/\lambda \alpha \right\} - \exp \left\{ v^{(0,a')} (x)/\lambda \alpha \right\} \right\}
\]

\[
\left[ \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \exp \left\{ v^{(0,\tilde{a})} (x)/\lambda \alpha \right\} \right]^{2}
\]

\[
= \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \exp \left\{ v^{(0,\tilde{a})} (x)/\lambda \alpha \right\} \frac{\sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \exp \left\{ v^{(0,\tilde{a})} (x)/\lambda \alpha \right\} \left[ \frac{v^{(0,a')} (x)}{\lambda} - \frac{v^{(0,\tilde{a})} (x)}{\lambda} \right]}{\sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \exp \left\{ v^{(0,\tilde{a})} (x)/\lambda \alpha \right\}}
\]

\[
= \frac{1}{\lambda} \hat{\sigma}^{(0,a')} (x) \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \hat{\sigma}^{(0,\tilde{a})} (x) \left[ v^{(0,a')} (x) - v^{(0,\tilde{a})} (x) \right].
\]

We can sign this derivative according to

\[
\frac{\partial \hat{\sigma}^{(0,a')} (x)}{\partial (1/\alpha)} > 0 \iff \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \hat{\sigma}^{(0,\tilde{a})} (x) \left[ v^{(0,a')} (x) - v^{(0,\tilde{a})} (x) \right] > 0
\]

\[
\iff v^{(0,a')} (x) > \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \hat{\sigma}^{(0,\tilde{a})} (x) v^{(0,\tilde{a})} (x)
\]

where the second line uses the fact that \(\sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \hat{\sigma}^{(0,\tilde{a})} (x) = 1\) by construction. Therefore, the probability of choosing \((0, a')\) conditional on not defaulting increases in \(1/\alpha\) (decreases in \(\alpha\)) if and only if the conditional value of choosing \((0, a')\), \(v^{(0,a')} (x)\), exceeds the expected value of choosing from the set of alternative actions \((0, \tilde{a})\) at the current decision rule. A symmetric calculation reveals that

\[
\frac{\partial \hat{\sigma}^{(0,a')} (x)}{\partial (1/\lambda)} = \frac{1}{\lambda} \hat{\sigma}^{(0,a')} (x) \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \hat{\sigma}^{(0,\tilde{a})} (x) \left[ v^{(0,a')} (x) - v^{(0,\tilde{a})} (x) \right]
\]

\[
\implies \frac{\partial \hat{\sigma}^{(0,a')} (x)}{\partial (1/\lambda)} > 0 \iff v^{(0,a')} (x) > \sum_{(0,\tilde{a}) \in \mathcal{F}(x)} \hat{\sigma}^{(0,\tilde{a})} (x) v^{(0,\tilde{a})} (x)
\]

Therefore, with respect to choice probabilities conditional on not defaulting, the effects of both extreme value parameters are symmetric.

Default actions  Equation (10) defines the probability of default as a function of the conditional value of the default action and the inclusive value of not defaulting, \(W_{ND} (x)\), which takes the

\(^{35}\)This keeps the analysis clean by avoiding repeated applications of the quotient rule for derivatives to the extent possible.
familiar log-sum form of (9). Since it will be useful in computing how \( \sigma^{(1,0)}(x) \) varies with \( \alpha \) and \( \lambda \), we can begin by taking derivatives of \( W_{ND} \):

\[
\frac{\partial W_{ND}(x)}{\partial (1/\alpha)} = \alpha \frac{\sum_{(0,a') \in F(x)} \exp \left\{ v^{(0,a')}(x)/\lambda \alpha \right\} \sigma^{(0,a')}(x)}{\sum_{(0,a') \in F(x)} \exp \{ v^{(0,a')}(x)/\lambda \alpha \}}
\]

\[
-\alpha^2 \log \left( \sum_{(0,a') \in F(x)} \exp \left\{ v^{(0,a')}(x)/\lambda \alpha \right\} \right)
\]

\[
= \alpha \lambda \sum_{(0,a') \in F(x)} \sigma^{(0,a')}(x)v^{(0,a')}(x) - \alpha W_{ND}(x)
\]

\[
\frac{\partial W_{ND}(x)}{\partial (1/\lambda)} = \alpha \frac{\sum_{(0,a') \in F(x)} \exp \left\{ v^{(0,a')}(x)/\alpha \right\} \sigma^{(0,a')}(x)}{\sum_{(0,a') \in F(x)} \exp \{ v^{(0,a')}(x)/\alpha \}}
\]

\[
= \sum_{(0,a') \in F(x)} \sigma^{(0,a')}(x)v^{(0,a')}(x).
\]

Returning to the default decision,

\[
\frac{\partial \sigma^{(1,0)}(x)}{\partial (1/\alpha)} = \left\{ \exp \left\{ v^{(1,0)}(x)/\alpha \right\} v^{(1,0)}(x) \left( \exp \left\{ v^{(1,0)}(x)/\alpha \right\} + \exp \{ \lambda W_{ND}(x)/\alpha \} \right) \right\}
\]

\[
- \exp \left\{ v^{(1,0)}(x)/\alpha \right\} \left( \exp \left\{ v^{(1,0)}(x)/\alpha \right\} v^{(1,0)}(x) + \exp \{ \lambda W_{ND}(x)/\alpha \} \lambda \frac{\partial W_{ND}(x)}{\partial (1/\alpha)} \right) \}
\]

\[
/ \left[ \left( \exp \left\{ v^{(1,0)}(x)/\alpha \right\} + \exp \{ \lambda W_{ND}(x)/\alpha \} \right) \right]^2
\]

\[
= \frac{\exp \left\{ v^{(1,0)}(x)/\alpha \right\} \exp \{ \lambda W_{ND}(x)/\alpha \}}{\left[ \exp \left\{ v^{(1,0)}(x)/\alpha \right\} + \exp \{ \lambda W_{ND}(x)/\alpha \} \right]^2} \left[ v^{(1,0)}(x) - \lambda \frac{\partial W_{ND}(x)}{\partial (1/\alpha)} \right]
\]

\[
= \sigma^{(1,0)}(x) \left( 1 - \sigma^{(1,0)}(x) \right) \left[ v^{(1,0)}(x) - \alpha \sum_{(0,a') \in F(x)} \sigma^{(0,a')}(x)v^{(0,a')}(x) + \alpha \lambda W_{ND}(x) \right]
\]

which is positive if and only if

\[
\frac{v^{(1,0)}(x)}{\alpha} > \sum_{(0,a') \in F(x)} \sigma^{(0,a')}(x)v^{(0,a')}(x) - \lambda W_{ND}(x).
\]

Therefore, as \( \alpha \) decreases, the probability of default increases if and only if the conditional value of defaulting, scaled by \( 1/\alpha \), exceeds the expected value of all non-default actions less the inclusive value of not defaulting. In the case when \( \lambda = 1 \) (i.e. when our nested logit structure collapses to standard logit), this condition reduces to evaluating the default action in the same way as the non-default actions above.
Likewise, turning to the correlation parameter $\lambda$,

$$
\frac{\partial \sigma^{(1,0)}(x)}{\partial (1/\lambda)} = - \exp\left\{ v^{(1,0)}(x)/\alpha \right\} \exp\{\lambda W_{ND}(x)/\alpha\} \left[ \frac{\lambda}{\alpha} \frac{\partial W_{ND}(x)}{\partial (1/\lambda)} - \frac{\lambda^2}{\alpha} W_{ND}(x) \right] \\
\left\{ \left[ \exp\left\{ v^{(1,0)}(x)/\alpha \right\} + \exp\{\lambda W_{ND}(x)/\alpha\} \right]\right\}^2 \\
= \sigma^{(1,0)}(x) \left( 1 - \sigma^{(1,0)}(x) \right) \left[ \lambda W_{ND}(x) - \sum_{(0,a') \in \mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x) \right]
$$

This implies that

$$
\frac{\partial \sigma^{(1,0)}(x)}{\partial (1/\lambda)} > 0 \iff \lambda W_{ND}(x) > \sum_{(0,a') \in \mathcal{F}(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x),
$$

i.e. if a scaling of the inclusive value of not defaulting exceeds the expected value of not defaulting.

Finally,

$$
\arg \max_{(d,a') \in \mathcal{F}(x)} \sigma^{(d,a')}(x) = \arg \max_{(d,a') \in \mathcal{F}(x)} v^{(d,a')}(x),
$$

so that the action which delivers the highest total utility before the extreme value shock is chosen with the highest probability. Combining these two pieces of information, we see that as $\alpha$ increases, the probability of choosing the action with the highest conditional value increases relative to all other feasible actions. Put differently, as $\alpha$ increases, (i) more and more weight is placed on the modal action, and (ii) the mean action converges to the modal action. For actions that are “suboptimal” in the sense that they deliver lower conditional value than the modal action, the change in weight depends on the difference in conditional value, weighted by the total value of these actions. This can have a meaningful impact on the mean action taken, if not the mode, which can effect prices and type scores significantly.

### B.2.2 Changing EV shocks in the full information model

Figure 20 demonstrates the impact of changing $\alpha$ and $\lambda$ on decisions in the full information model, discussed in Section B.7.1 below. Each figure contains three lines, corresponding to: (i) the baseline parameterization of Table 3 (black solid); (ii) a “low variance” parameterization (blue dotted) where $\alpha$ is reduced by 75% and $\lambda$ is held fixed; and (iii) a “high correlation” parameterization (red dashed) in which $\lambda$ is reduced by 75% and $\alpha$ is held fixed. All figures are presented for an agents with $(\beta, e, z) = (\beta_H, 1, 0)$. In each parameterization, the equilibrium pricing function, and therefore the conditional action values, are held fixed, and so the changes in response shown here can be thought of as partial equilibrium in order to highlight the direct effects on decisions.

Consider first the default decision. The top left panel shows how the default decision varies over a range of levels of debt. By lowering the variance, the slope of increase in the probability of default as the level of indebtedness increases is much sharper than in the baseline parameterization. This is because there is less chance for a high value shock to be realized for an action with lower fundamental value, so the decision rule becomes more centered at the mode for each level of $a$. By raising the correlation, the expected value of not defaulting is reduced, and so the default decision rule “shifts to the right,” i.e. agents default more frequently for lower levels of debt.

The remaining three figures show how non-default decisions are affected by changes in the extreme value parameters. The top right panel depicts the modal decision across each case (with
Figure 20: Impact of extreme value preference shocks

Notes: Baseline refers to the parameterization of the extreme value shock process from Table 3. Low \( \alpha \) (\( \lambda \)) is a quarter of the baseline value: \( \alpha' = \alpha/4 \ (\lambda' = \lambda/4) \). All panels are constructed from the full information model, and fix the state of an agent at \((\beta, e, z) = (\beta_H, 1, 0)\).

default depicted as choosing \( a' = -1 \) for simplicity). Since the action values are held fixed, according to the analysis above the only parameter which can induce a change in the modal decision is the correlation parameter \( \lambda \), and even this change can only apply to the default / no default decision. The top right panel confirms this.

The bottom left and bottom right panels show the mean and standard deviation of the decision rule, respectively. The actions of zero to the left of the figure reflect default. The major changes across parameterizations occur when default is an option, i.e. when \( a < 0 \). In this case, we see a steeper shift in the mean decision for low variance, consistent with the analysis above. Additionally, we see that the actions are biased towards default with a higher correlation. Turning to the standard deviation in choices, both alternatives feature lower dispersion in decision rules than our baseline economy. In the low variance economy, that dispersion is the lowest of the three cases, and reaches its peak at the level of assets under which the default decision is “interior.” In the high correlation economy, there is similarly a smaller range of default decisions and more centering around non-default choices.

B.3 Grids

Table 6 presents the key grids used in the computational analysis. Note in particular that the asset and type score grids are quite dense in order to insure convergence, while in contrast the earnings and type grids are coarse in order to ease the computational burden and simplify the analysis.
Table 6: Grids used in computational analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>$x$</th>
<th>$N(x)$</th>
<th>Range / Values</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>2</td>
<td>${0.915, 0.886}$</td>
<td>2-point support makes Bayesian functions scalar-valued.</td>
</tr>
<tr>
<td>Earnings (per)</td>
<td>$e$</td>
<td>3</td>
<td>${0.57, 1.00, 1.74}$</td>
<td>See Table 1</td>
</tr>
<tr>
<td>Earnings (trans)</td>
<td>$z$</td>
<td>3</td>
<td>${-0.18, 0.00, 0.18}$</td>
<td>See Table 1</td>
</tr>
<tr>
<td>Assets</td>
<td>$a$</td>
<td>150</td>
<td>$[-0.25, 15.00]$</td>
<td>50 points in neg. region, 100 in pos. Density close to 0 is critical.</td>
</tr>
<tr>
<td>Type score</td>
<td>$s$</td>
<td>50</td>
<td>$[0.0, 1.0]$</td>
<td>bounded below by low $\beta$ to high $\beta$ transition, above by high to low.</td>
</tr>
</tbody>
</table>

B.4 Model moment definitions

**Default rate** The default rate is computed as the total fraction of the population who defaults within a given period. The probability of a given state is given by $\mu(\cdot)$, and the probability of default given a state is $\sigma^{(1,0)}(\cdot)$, and so the aggregate default rate is

$$\text{aggregate default rate} = \sum_{\beta,z,\omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega).$$

By type, we have $\sum_\omega \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega) / \sum_\omega \mu(\beta, z, \omega)$

**Fraction in debt** This is the fraction of the population with $a < 0$ in a given period, so

$$\text{fraction in debt} = \sum_{\beta, z, \omega | a < 0} \mu(\beta, z, \omega).$$

By type, the analogous figure is $\sum_{\omega | a < 0} \mu(\beta, z, \omega) / \sum_\omega \mu(\beta, z, \omega)$

**Median net worth to median income** This is the ratio of the relevant medians, computed with respect to the stationary distribution.

**Debt to income** Income in the model is given by the sum of earnings (persistent and transitory) and net interest on assets. That is, income $= e + z + (1/q(a', p) - 1) \cdot a$. Therefore, debt to income is computed as the weighted average of the ratio of assets, $a$, to income conditional on $a$ being negative:

$$\text{debt to earnings} = \sum_{\beta, z, \omega | a < 0} \frac{a}{e + z + (1/q - 1) \cdot a} \cdot \frac{\mu(\beta, z, \omega)}{\sum_{\beta, z, \omega | a < 0} \mu(\beta, z, \omega)}.$$

**Average interest rate** The average interest paid (or received) by the agents in the economy is the weighted average of the interest rates paid, $1/q - 1$, over the stationary distribution and decision probabilities.

$$\text{average interest rate} = \sum_{\omega} \mu(\omega) \cdot \sum_{\beta, z} \frac{\mu(\beta, z, \omega)}{\mu(\beta, z, \omega)} \cdot \sum_{a'} \frac{\sigma^{(0,a')}(\beta, z, \omega)}{\sum_a \sigma^{(0,a')}(\beta, z, \omega)} \left( \frac{1}{q^{(0,a')}(\omega)} - 1 \right).$$
where \( \mu(\omega) = \sum_{\beta, z} \mu(\beta, z, \omega) \).

**B.5 Credit Access Following Default**

For all \( a' < 0 \), define the two price schedules

\[
q_D^{(0,a')}(e, a, s) \equiv q^{(0,a')}(e, 0, \psi(1,0)(e, a, s)),
\]
\[
q_N^{(0,a')}(e, a, s) \equiv q^{(0,a')}(e, 0, \psi(0,0)(e, a, s)),
\]

where the former corresponds to default (\( D \)) and the latter corresponds to no default (\( N \)). Further let \( q_0^{(0,a')}(e, a, s) \) denote the price schedule for an agent in state \((e, a, s)\) in the period before default. These three schedules – \( q_0^{(0,a')}, q_D^{(0,a')}, \) and \( q_N^{(0,a')} \) – can be compared for each \((e, a, s)\).

In order to compute an “average” effect of defaulting, we can weight the price differences for each action by the stationary distribution of agents who have the option to default. Specifically, define

\[
\bar{\mu}(e, a, s) = \frac{\sum_{\beta, z} \mu(\beta, e, z, a, s)}{\sum_{\beta, z, \bar{a} < 0} \mu(\beta, e, z, \bar{a}, s)} \quad \text{for all } a < 0.
\]

Then, we can compute the aggregate metrics for each debt choice \( a' < 0 \)

\[
\Delta_D^{q}(a') = \sum_{e, a < 0, s} \bar{\mu}(e, a, s) \left[ q_D^{(0,a')}(e, a, s) - q_D^{(0,a')}(e, a, s) \right],
\]
\[
\Delta_N^{q}(a') = \sum_{e, a < 0, s} \bar{\mu}(e, a, s) \left[ q_N^{(0,a')}(e, a, s) - q_N^{(0,a')}(e, a, s) \right].
\]

**B.6 Welfare Metric**

First, recognize that for a given state \((\beta_t, x)\), the value of an agent is

\[
V(\beta_t, x) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \rho^t B_t \frac{c_t^{1-\gamma} - 1}{1 - \gamma} \right],
\]

where \( B_0 = 1 \), and for all \( t \geq 1 \)

\[
B_t = \prod_{i=0}^{t-1} \beta_i,
\]

so that \( B_1 = \beta_0, B_2 = \beta_0 \beta_1, \) etc.

Let an agent’s current \( \beta \) be given, and let \( W \) be an agent’s value under an alternative market arrangement. Let \( x \) capture all other state variables besides \( \beta \). Then, the consumption equivalent
measure is defined by

\[ W(\beta, x) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \rho^t B_t \frac{\left(1 + \lambda(\beta, x)\right) c_t^{1-\gamma} - 1}{1 - \gamma} \right] = (1 + \lambda(\beta, x))^{1-\gamma} \mathbb{E} \left[ \sum_{t=0}^{\infty} \rho^t B_t \frac{c_t^{1-\gamma}}{1 - \gamma} \right] - \mathbb{E} \left[ \sum_{t=0}^{\infty} \rho^t B_t \right] \]

\[ \equiv A(\beta) \]

\[ \implies W(\beta, x) + A = (1 + \lambda(\beta, x))^{1-\gamma} \left( \mathbb{E} \left[ \sum_{t=0}^{\infty} \rho^t B_t \frac{c_t^{1-\gamma} - 1}{1 - \gamma} \right] + A \right) \]

\[ \implies 1 + \lambda(\beta, x) = \left[ \frac{W(\beta, x) + A}{1 - \gamma} \right]^{1-\gamma} \]

The only difference relative to the constant \( \beta \) case is that \( \frac{1}{(1-\gamma)(1-\beta)} \) is replaced by the more subtle term \( A(\beta) \). To the best of our knowledge, this term does not admit a closed form. However, it can be computed by simulating \( N \) paths of length \( T \) of \( \beta \) starting from a given \( \beta_0 \), \( \{\beta_0(n), ..., \beta_T(n)\}_{n=1}^{N} \). Pick \( T \) sufficiently large that \( \rho^t \) becomes vanishingly small. For each path \( n \), compute \( \{B_0(n), ..., B_T(n)\} \). Then, compute

\[ \hat{A}(\beta) = \frac{1}{N} \sum_{n=1}^{N} \left( \sum_{t=0}^{T} \rho^t B_t(n)/(1 - \gamma) \right) \]

**B.7 Details of Alternative Economies**

**B.7.1 Full information (FI)**

Since there is no incentive to infer one’s type, there is no type score in this model. Therefore, an agent’s full state is \((\beta, e, z, a)\), and the set of equilibrium functions does not include \( \psi \). For comparability, and since it is purely i.i.d. and contains no information for inference, we maintain the assumption that \( z \) is unobservable. Therefore, the lender can observe \( \omega_{FI} = (\beta, e, a) \) for each individual.

The household problem and equilibrium stationary distribution are exactly the same as in the main text, with the state variable \( s \) removed. The only substantial change is in the pricing and repayment probability equations. The repayment probability function in this case is \( p_{FI}^{(0,a')}(\omega_{FI}) = \text{Pr}(\text{repay } a'|\omega_{FI}) \). Since \( \omega_{FI} \) directly includes \( \beta \) and \( z \) is i.i.d., there is no further inference to be done. Therefore, \( a \) has no impact on pricing, and we obtain

\[ p_{FI}^{(0,a')} (\beta, e) = \sum_{\beta', e', z'} \left[ 1 - \sigma_{FI}^{(1,0)} (\beta', e', z', a') \right] Q^\beta (\beta'|\beta) Q^e (e'|e) H(z'). \]  \hfill (37)

The loan pricing function, \( q_{FI}^{(0,a')} (\beta, e) \), simply adjusts the expression for the interest rate as in the baseline model.

**B.7.2 No tracking (NT)**

The key departure in the formal specification of this economy from the baseline comes from the specification of the type score updating function, \( \psi_{NT} \). Here, the assessment of types updates only...
based on the exogenous transition probabilities, and so there is no dynamic incentive to acquire reputation. As a result,
\[ \psi_{NT,\beta}(s) = \sum_{\beta} Q^\beta(\beta'|\beta)s(\beta). \]

In the two-type case we employ in our quantitative model, we have
\[ \psi_{NT}(s) = sQ^\beta(\beta_H|\beta_H) + (1 - s)Q^\beta(\beta_H|\beta_L). \] (38)

Loan pricing occurs in a way similar to the baseline economy, and the observable state of an individual is still \( \omega_{NT} = \omega = (e,a,s) \). Recall, however, that the only reason prices depend on current assets in the baseline economy is that \( a \) is an argument of the \( \psi \) function, and repayment probabilities are impacted by an agent's assessed type tomorrow, \( s' = \psi(d,a')(e,a,s) \). In this case without inference, however, \( s' = \psi_{NT}(s) \), and so there is no induced dependence on \( a \). Therefore, we obtain the repayment probability expression
\[ p_{NT}^{(0,a')}(e,s) = \sum_{\beta,\beta',e',z'} \left[ 1 - \sigma^{(1,0)}(\beta',e',z',a',\psi_{NT}(s)) \right] s(\beta)Q^\beta(\beta'|\beta)Q^e(e'|e)H(z'). \] (39)

Again, the loan pricing function \( q_{NT}^{(0,a')}(e,s) \) simply adjusts the expression above for the rate of interest.

### B.7.3 Credit Score Interactions Across Markets

In our baseline model, one's type score (or reputation) only plays a role by affecting unsecured debt prices. In reality, though, credit scores impact more than credit card interest rates. Financially, they impact mortgage rates, auto loan rates, and many other forms of credit; in other areas, they can affect employment outcomes. In this section, we consider in a simplified way how these interactions affect the acquisition of reputation and behavior in the unsecured credit market.

To capture the role that credit scores play across markets (AM), we assume that each period an individual must finance an expenditure weakly calibrated to proxy mortgage payments on a home loan. This expenditure is decreasing in type score, which proxies the empirical fact that individuals with higher credit scores tend to pay lower interest rates on standard mortgage contracts. All of the equilibrium type scoring and pricing functions are specified in exactly the same way as in the baseline economy; the only thing that differs is the household’s budget constraint. Specifically, we model the financial burden of a bad reputation as an additional expense
\[ m(s) = m_0 - m_1 s. \] (40)

The household’s flow income becomes \( e + z - m(s) \), as opposed to simply \( e + z \).\(^{36}\)

The baseline results are collected in the last column (AM) of Table 7. We see that the effects of modeling reputation in this way are large. Relative to the baseline economy, the default rate drops by 12%. While average interest rates remain close to their baseline levels, dispersion in interest rates rises, and quantities of debt decline sharply on both the extensive and intensive margins. There are three sharp changes in the credit ranking life cycle moments. First, the intercept of the lifetime path of credit rankings is much higher in the AM economy: the only thing that differs is the household’s budget constraint. Specifically, we model the financial burden of a bad reputation as an additional expense

\[^{36}\]The parameters \( m_0 \) and \( m_1 \) in equation (40) are chosen to be consistent with a scaled down version of the cost of holding a 30 year fixed-rate mortgage on a home of median price as of 2007.
Table 7: Comparison between Baseline and Across Market (AM) Economies

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Subgroup</th>
<th>Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BASE</td>
<td>AM</td>
</tr>
<tr>
<td>Aggregate credit market statistics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>default rate</td>
<td>aggregate</td>
<td>0.975</td>
<td>0.915</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>0.712</td>
<td>0.662</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>1.181</td>
<td>1.099</td>
</tr>
<tr>
<td>average interest rate</td>
<td>aggregate</td>
<td>13.95</td>
<td>13.84</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>14.62</td>
<td>14.71</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>13.42</td>
<td>13.21</td>
</tr>
<tr>
<td>interest rate dispersion</td>
<td>aggregate</td>
<td>7.240</td>
<td>8.595</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>6.079</td>
<td>9.058</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>7.994</td>
<td>8.184</td>
</tr>
<tr>
<td>fraction of HH in debt</td>
<td>aggregate</td>
<td>10.50</td>
<td>9.12</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>8.222</td>
<td>6.988</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>12.29</td>
<td>10.68</td>
</tr>
<tr>
<td>debt to income ratio</td>
<td>aggregate</td>
<td>0.251</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>0.190</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>0.300</td>
<td>0.271</td>
</tr>
<tr>
<td>Credit ranking age profile moments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>intercept, mean credit ranking</td>
<td></td>
<td>0.355</td>
<td>0.424</td>
</tr>
<tr>
<td>slope, mean credit ranking</td>
<td></td>
<td>0.029</td>
<td>0.018</td>
</tr>
<tr>
<td>intercept, std. dev. credit ranking</td>
<td></td>
<td>0.255</td>
<td>0.240</td>
</tr>
<tr>
<td>slope, std. dev. credit ranking</td>
<td></td>
<td>0.004</td>
<td>0.008</td>
</tr>
<tr>
<td>average autocorr of change in credit ranking</td>
<td></td>
<td>-0.109</td>
<td>-0.143</td>
</tr>
</tbody>
</table>

to acquire a good reputation, young agents are in general much less risky in the AM economy than in the baseline. Second, the average improvement in credit ranking over one’s lifetime is much lower, dropping by more than 1/3 relative to the baseline. Third, the mean reversion in credit rankings is much sharper in the AM economy. Given the additional costs of having a bad reputation, agents tend to more sharply “correct” past actions which were destructive of their credit scores.

B.8 Sensitivity Analysis

We further explore the effect of our parameters on our target moments by solving the model changing each parameter by a small amount individually. The aggregate credit market statistics resulting from increasing each parameter by 5% in turn are reported in Table 8.

Increasing $\beta_H$ increases the “distance” between types. High types default slightly less, while low types default 21% more because it is more costly for them to imitate high types, and so the aggregate default rate rises. The average interest rate paid in aggregate rises sharply because of the
### Table 8: Sensitivity Analysis for Credit Market Moments

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Base.</th>
<th>Sensitivity to 5% increase in $\beta_H$</th>
<th>$\beta_L$</th>
<th>$Q^\beta_{HL}$</th>
<th>$Q^\beta_{LH}$</th>
<th>$\lambda$</th>
<th>$\alpha$</th>
<th>$G_{\beta_H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate credit statistics</strong></td>
<td></td>
<td></td>
<td>$\beta_H$</td>
<td>$\beta_L$</td>
<td>$Q^\beta_{HL}$</td>
<td>$Q^\beta_{LH}$</td>
<td>$\lambda$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>default rate</td>
<td>agg.</td>
<td>0.98</td>
<td>1.12</td>
<td>0.93</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>0.71</td>
<td>0.70</td>
<td>0.82</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>1.18</td>
<td>1.43</td>
<td>1.01</td>
<td>1.17</td>
<td>1.18</td>
<td>1.18</td>
<td>1.22</td>
</tr>
<tr>
<td>average int. rate</td>
<td>agg.</td>
<td>13.95</td>
<td>15.91</td>
<td>14.32</td>
<td>13.95</td>
<td>13.96</td>
<td>14.09</td>
<td>15.08</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>14.62</td>
<td>19.63</td>
<td>14.06</td>
<td>14.64</td>
<td>14.64</td>
<td>14.81</td>
<td>15.96</td>
</tr>
<tr>
<td>int. rate dispersion</td>
<td>agg.</td>
<td>7.24</td>
<td>10.68</td>
<td>6.31</td>
<td>7.25</td>
<td>7.25</td>
<td>7.23</td>
<td>7.52</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>6.08</td>
<td>12.02</td>
<td>6.12</td>
<td>6.08</td>
<td>6.09</td>
<td>6.07</td>
<td>6.50</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>7.99</td>
<td>8.66</td>
<td>7.98</td>
<td>7.99</td>
<td>7.98</td>
<td>8.14</td>
<td>7.96</td>
</tr>
<tr>
<td>fraction HH in debt</td>
<td>agg.</td>
<td>10.50</td>
<td>9.74</td>
<td>9.62</td>
<td>10.52</td>
<td>10.52</td>
<td>10.52</td>
<td>10.72</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>8.22</td>
<td>6.32</td>
<td>8.51</td>
<td>8.27</td>
<td>8.27</td>
<td>8.25</td>
<td>8.47</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>12.29</td>
<td>12.20</td>
<td>10.39</td>
<td>12.23</td>
<td>12.26</td>
<td>12.27</td>
<td>12.41</td>
</tr>
<tr>
<td>debt / income ratio</td>
<td>agg.</td>
<td>0.25</td>
<td>0.26</td>
<td>0.24</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>high $\beta$</td>
<td>0.19</td>
<td>0.18</td>
<td>0.21</td>
<td>0.19</td>
<td>0.19</td>
<td>0.19</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>low $\beta$</td>
<td>0.30</td>
<td>0.32</td>
<td>0.26</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Credit score life cycle</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>intercept, mean</td>
<td></td>
<td>0.35</td>
<td>0.35</td>
<td>0.24</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.36</td>
</tr>
<tr>
<td>slope, mean</td>
<td></td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>intercept, std. deviation</td>
<td></td>
<td>0.25</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td>slope, std. deviation</td>
<td></td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>average, autocorrelation</td>
<td></td>
<td>-0.11</td>
<td>-0.10</td>
<td>-0.13</td>
<td>-0.11</td>
<td>-0.11</td>
<td>-0.13</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

Inference problem: even though high types default less, they face higher prices on average because the low types behave so much worse and the types cannot be perfectly separated. Correspondingly, the fraction of high types in debt decreases, and despite an increase in indebtedness by low types, the mass of debtors in the economy decreases overall.

In contrast, increasing $\beta_L$ lowers this between-type distance. High types default more relative to the baseline, while low types default less. Both outcomes hinge on reputation; high types have less incentive to separate themselves from low types through good behavior, while low types find it more attractive to imitate high types and enhance their reputations. Notably, this economy features a large incentive to avoid debt and the reputational hit associated with it, as evidenced by the large declines in the debt to income ratio and fraction of households in debt.

Changes to the type transition process $Q^\beta$ have similar, but smaller effects. Increasing $Q^\beta_{HL}$ lowers the stationary fraction of high types, which is analogous to increasing $\beta_L$ or decreasing $\beta_H$. This is because high types in this economy have a lower expected future $\beta$, and therefore effectively discount the future more steeply. The reverse holds for increasing $Q^\beta_{LH}$.

Lastly, increasing $\alpha$ increases the overall variance of the extreme value shocks. As actions become noisier, default rates increase for both types. Interest rates follow the same pattern, but critically agents do not react to these increased interest rates by borrowing less: both the fraction
of households in debt and the debt to income ratio increase. Fixing the scale parameter and varying \( \lambda \) instead has a markedly different effect. While the default vs. no default decision remains as noisy as in the baseline case, the choice of \( a' \) effectively becomes less noisy: since the shocks for specific \( a' \) are now less correlated, it is more likely that an action with low conditional value will receive a high enough \( \epsilon \) shock to merit choosing it over an action with high conditional value. Therefore, agents choose actions with a lower “bang-for-buck” and the average interest rate rises due to this selection effect.

C Data Appendix

C.1 Data on Credit Scores

This appendix describes the construction of the data underlying the life cycle credit score moments reported in Table 2 and credit score pattern shown for bankrupts in Figure 3.

Turning first to the construction of the data for life cycle moments, we began with a 2 percent random sample of the FRBNY/CCP panel containing each individual’s birth year and the individual’s credit score in the first quarters of 2004, 2005 and 2006. We drop individuals who are not within the ages of 21 and 60 years in 2004Q1 and any individual whose credit score is missing for any of the quarters. This yields our base sample. For this sample, we constructed empirical cumulative distribution functions of credit scores for each of the three years. And, for each individual, we computed the percentile ranking of his or her score for each year. We call this the individual’s score percentile — it is a number that gives the fraction of people who had credit scores not exceeding the individual’s score in that year. We then placed individuals in 5-year age bins according to their age in 2004Q1. Table 9 reports the mean and standard deviations of the score percentiles in each bin. These moments were used in the regressions that determine the coefficients in the first 5 rows of the bottom panel of Table 2. For the final two rows, we computed the change in an individual’s credit score percentile from 2004Q1 to 2005Q1 and, again, from 2005Q1 to 2006Q1. For each age bin, we computed the correlation between these pair of changes across individuals in that bin. These correlations are reported in the final column of Table 9. The final two rows of Table 2 in the main text reports the age pattern in these correlations.

Turning next to the construction of the data for the default event study, we first isolated about 50,000 individuals who filed for Chapter 7 bankruptcy in sometime in 2004 and obtained a discharge sometime in 2005. As above, we eliminated all individuals younger than 21 years and older than 60 years. This yielded our base sample of bankrupts. For each individual in this sample, we recorded their birth year and credit score in the filing quarter and in the four quarters preceding and following the filing quarter. We placed each individual in the appropriate 5-year age bin based on their age in 2004. We computed the average credit score in each age bin for each of the nine quarterly observations and then computed the fraction of individuals in the base sample who had scores not exceeding this average. This gave us the average score percentile of people in the base sample of bankrupts reported in Table 10.
Table 9: Life Cycle Credit Score Moments

<table>
<thead>
<tr>
<th>Age Bins</th>
<th>Mean, Score Pctl</th>
<th>SD, Score Pctl</th>
<th>Corr(ΔPctl_{04}, ΔPctl_{05})</th>
</tr>
</thead>
<tbody>
<tr>
<td>21-25 years</td>
<td>0.32</td>
<td>0.20</td>
<td>-0.22</td>
</tr>
<tr>
<td>26-30 years</td>
<td>0.35</td>
<td>0.23</td>
<td>-0.18</td>
</tr>
<tr>
<td>31-35 years</td>
<td>0.40</td>
<td>0.26</td>
<td>-0.20</td>
</tr>
<tr>
<td>36-40 years</td>
<td>0.44</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>41-45 years</td>
<td>0.47</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>46-50 years</td>
<td>0.50</td>
<td>0.28</td>
<td>-0.21</td>
</tr>
<tr>
<td>51-55 years</td>
<td>0.54</td>
<td>0.28</td>
<td>-0.19</td>
</tr>
<tr>
<td>56-60 years</td>
<td>0.58</td>
<td>0.28</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

Table 10: Default event study: raw data

<table>
<thead>
<tr>
<th>Quarters</th>
<th>26-30 years</th>
<th>31-35 years</th>
<th>36-40 years</th>
<th>41-45 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>0.23</td>
<td>0.21</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>-3</td>
<td>0.19</td>
<td>0.18</td>
<td>0.17</td>
<td>0.15</td>
</tr>
<tr>
<td>-2</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>-1</td>
<td>0.13</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>0</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.22</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>0.28</td>
<td>0.25</td>
<td>0.24</td>
<td>0.21</td>
</tr>
<tr>
<td>3</td>
<td>0.28</td>
<td>0.26</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>4</td>
<td>0.28</td>
<td>0.26</td>
<td>0.25</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Notes: The data presented in these table corresponds to the black lines in Figure 3.
Figure 21: Credit score life cycle moments: model vs. data

Notes: The model and data moments in these plots are the raw profiles underlying the credit score life cycle moments (regression coefficients) in Table 2. The raw data are in Table 9.